USING HABERMAS TO EXPLAIN WHY LOGICAL GAMES FOSTER ARGUMENTATION

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Argumentation and proof have been in the focus of attention in mathematics education research for several decades. While it has often been pointed out that it is important to give argumentation a prominent position in the mathematics classroom, it is far from clear how to teach argumentation, particularly to students from non-privileged backgrounds. In this paper, I show how Habermas' theory of communicative action gives a valuable perspective on what makes argumentation likely to occur. The context of a logical game situation in which argumentation happened is analysed to support the following result: the exclusion of force and a cooperative mode of communication are helpful elements in understanding the fostering of argumentation.

LEARNING ARGUMENTATION IS DOING ARGUMENTATION

Research in the past decades has looked at argumentation from various angles, often connected to mathematical proof. In this paper, Knipping's (2003, p. 34) view is adopted: argumentation is seen as a sequence of utterances in which a claim is made and reasons are brought forth to rationally support this claim; so proof is one form of argumentation. Different approaches have been made to promote proving in the mathematics classroom[1]. Boero (2011, p. 120, italics in original) claims that to teach the rules of argumentation and proving: "the best didactical choice is to exploit *suitable mathematical activities* of argumentation and proof". Douek (1999) however pointed out that having proof as the goal of the activity can be a restraint for argumentation. I decided to look purely at argumentation and the question of how to involve students with a non-academic family background in reasoning.

The only way of learning argumentation is engaging in argumentation, and as Ernest (1986, p. 3) pointed out, "playing games demands involvement". In this theoretical paper I show how Habermas' threefold approach to argumentation from his theory of communicative action provides a useful perspective for looking at classroom situations, and how logical games used in the context of mathematics teaching can provide a fruitful environment for argumentation. I support this approach with an example from my research in which I use a game to involve my students in argumentation. After playing two rounds of the logical game "Da Vinci Code" in a competitive mode, the students were faced with a hypothetical situation based on the game, whose solution required deductive reasoning. A part of the students'

[1] Knipping (2012) presents a concise overview on different approaches to the teaching and learning of argumentation and proof, including graphical representations, the debate approach and the concept of cognitive unity.

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sophisticated argumentation is presented in this paper. I conclude by illuminating how mathematics education can benefit from Habermas' view on argumentation, and how logical games can provide a context to promote argumentation.

"IN THE MIDDLE, THERE IS TWO AND FIVE"

In my doctoral research, I worked for an entire year as a teacher-researcher in a group of five 15-year-old girls from different schools in Bremen, whose mother tongue is not German. In different learning situations, some purely mathematical and some including elements of logical games, I tried to evoke argumentation. The transcript given in this paper is an excerpt from a lesson in March 2013. The girls and I had known each other for 6 months. For the last lesson prior to the spring holidays I decided to pose a task related to a logical game called "Da Vinci Code", also known by the name "Coda" by Eiji Wakasugi. "Da Vinci Code" is about correctly guessing the numbers of your opponents. The game consists of black and white stones, 12 each, numbered from 0 to 11. At the beginning, each player takes a certain amount of stones and puts them up in front of him or herself, so that the other players cannot see the stones. They have to be put up in ascending order, and if a player has both stones of one number, the black number must stand left of the white number. In the course of the game, stones are taken up from the middle and wrong guesses lead to stones being tipped over, thereby revealing the number. At a certain point it can become possible from a player's perspective to correctly deduce all of the remaining stones.

I introduced the girls to the game in that lesson. There were only three girls present on that day, one of them does not make a contribution in the transcript; the others are labelled as S1 and S2. My contributions are labelled as "I". I translated the transcript from German to English as thoroughly as possible. The girls were allowed to play two rounds of the game before I took away the material and presented them with a fictional situation (cf. Figure 1) based on the game. The task for the girls was to find out all of the missing numbers; the only information they had was that all stones were arranged according to the rules of the game. I decided to work with a fictional situation so that the students could collaborate in finding a solution, in contrast to the game situation in which they were opponents. The transcript covers a time span of approximately 2 minutes, which took place directly after distributing the worksheets. In the situation, one of the girls (S2) argues that the two black stones in the middle need to be 2 and 5.

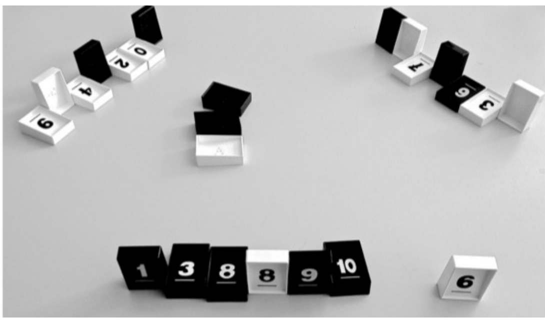

Figure 1: The fictional game situation

S1: (points to the stones in the middle) Do we have to guess these, too?
I: Yes, but of course there you cannot assign which one's which. But you will guess them in the end, so to say. Actually they are only those, which are left over.
S2: Hey, the two is IN THERE, in that, in that hole there.
S1: Mhm (agreeing; 27s silence)
I: (gets up) Feel free to tell the others when you found out a number. (11s)
S2: Well, in the middle there is five and two.
I: Mh (questioning), how do you think you know? (walks over)
S2: BECAUSE (1s), eh, the two, here (points to right opponent) it would not fit because there is a white one.
I: Mhm (agreeing).
S2: (4s) And here it would not fit (points to left opponent) because the black one isn't in front.
I: Yes (1s), that's true.
S2: Oh, and this is the three (points to the middle), isn't it?
I: No, the three is lying in front of you. I'm //not saying it's wrong//
S2: //No, I mean five, I mean five// and the five can't fit here (points to right opponent) because there is the six. And here, the five can't fit here (points to left opponent) because there is a four.
I: Yes (1s). Very nice. So the two and the five black are already set in the middle. (4s). Well considered. I actually thought they'd just be left over in the end (laughs).

After this situation, the girls found all other missing numbers with hardly any guidance and arrived at a correct solution for the overall situation in less than 10 minutes.

Analysis of the argumentation structure

In my analysis of the situation, I reconstructed the argumentation using the Toulmin scheme in the way Knipping (2008) introduced. The analysis is based on the transcript; the numbers in the boxes indicate the referenced lines. Implicit data and warrants are added for clarification, marked by dashed lines. Roman numbers indicate the three different warrants which occurred:

- I. All 24 stones (0 to 11, each once in white and once in black) are on the table, and there are no more stones than these.
- II. The stones are arranged in ascending order in front of the players.
- III. If a player has one number in both colours, the black stone stands left of the white stone.

In the scheme, data are represented as ellipses, both final and intermediate conclusions as rectangles, and warrants as diamonds. The implicit data is that the black 2 and the black 5 do not stand in front of the player; the blackened box stands for the false intermediate conclusion that the black 3 is in the middle.

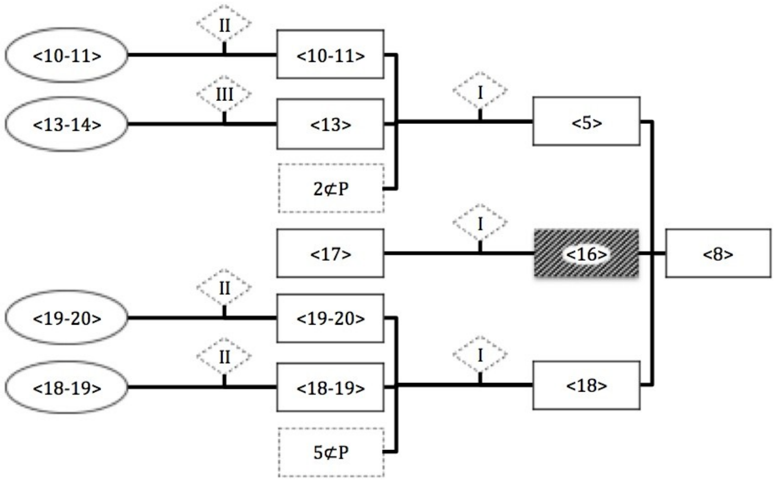

Figure 2: Logical analysis of the presented argument

The warrants used, not only in this transcript excerpt but also in all other arguments in that lesson, are equivalent to the rules of the game. In this argument, all warrants were left implicit which is common according to Toulmin (1958/2008). The structure of the argument is highly complex, and the only implicit parts are the warrants and the pieces of data referring to the immediately visible situation in front of the player.

Obviously, S2 was capable of using the rules learnt in the game to create a sophisticated argument. The deductions she makes to show that the black stones in the middle need to be 2 and 5 are similar to those used in mathematical proving. She comprehensibly establishes that both black 2 and black 5 have to be in the middle, for they cannot be in front of any of the players. In many other much less complex classroom situations, this particular student was not capable of creating arguments. This leads to the question, how argumentation was facilitated in the presented situation. In the following, I will elaborate a theoretical framework that can explain why logical games are likely to evoke argumentation and reasoning.

HABERMAS' THEORY OF COMMUNICATIVE ACTION

Boero (2006) introduced Habermas' concept of rationality into the analysis of argumentation and proving in mathematics education. This concept of rationality provides a fruitful tool for the analysis of argumentation and proving processes and their products. In this paper I use another concept from Habermas' theory of communicative action: the three-layered view on argumentation as a process, procedure and product. While Habermas' theory of rationality provides a tool for analysing the epistemic and cognitive aspects of actual argumentation and proving processes and products, the view on argumentation presented in this paper can provide an explanation why students do or do not engage in argumentation. In his theory of

communicative action, Habermas (1983) elaborates on how the sciences of rhetoric, dialectic and logic differ in their approaches to argumentation as processes, procedures and products.

Argumentation as a process

Rhetoric analyses focus on the process character of argumentation. From this perspective, Habermas (1983) describes argumentation as an act of structured communication that follows almost ideal preconditions. Characteristic for argumentation processes is the exclusion of force from outside and the reliance on nothing but the best arguments. The rules which, according to Habermas, any argumentation process needs to fulfil are: Every subject capable of speech and action may participate in the discourse; every participant may problematize and introduce any statement and utter his or her wishes, attitudes and needs; and no forces from within or without the discourse may hinder any participant to use these rights. While hardly any communicative situation objectively fulfils these criteria, Habermas clarifies that the subjective impression that these criteria are met is sufficient. The subjective feeling that there is no force from outside the situation is a prerequisite for engaging in argumentation.

School situations are usually marked by an imbalance in the distribution of power between teacher and students. The teacher controls and defines topics, suitable arguments, relevant background information and data that can be regarded as shared knowledge. For the students, it is often far from obvious which inference rules and data can be seen as common knowledge and where further clarification is required. Control remains with the teacher. Logical game situations, on the other hand, are shaped by clear instructions and equal positions of the participants. Although players may have a different level of experience, the game treats them as equal. Superiority can only arise from a better understanding of the instructions. The possibility of eye-level communication, the shared knowledge of rules and premises and the absence of force from outside create ideal preconditions for argumentation. In the situation presented in this paper, S2 self-confidently supports her claims with arguments. She clearly feels encouraged to engage in reasoning and to bring forth arguments to support her claims, and no force prevents her from doing so.

Argumentation as a procedure

Dialectic is the science concerned with argumentation as a procedure. Habermas (1981) characterizes argumentation procedures as cooperative communication situations in which proponents and opponents hypothetically check claims and their appropriateness by reasons, acting without pressure arising from experience or from a call to action. The rules for argumentation procedures (1983) are the following: Speakers are only allowed to claim what they believe, and if they attack statements or norms outside of the initial discussion matter, they need to give a reason. Arguments are the only way of reaching agreement, and cooperative communication of all participants is necessary to reach a decision.

In the mathematics classroom, students sometimes make claims without being convinced of their truth, looking for their teacher's evaluation of validity. In game situations, on the other hand, claims are made according to the players' best hypothesis, intrinsically motivated by the desire to win. If the rules of the game do not allow for any activity but hypothetically considering logical implications, reasons are the only available way of dealing with the situation. Topics which are outside of the game content are unlikely to be introduced into the discussion while playing a game, because the validity of the rules is strictly limited to the game setting. In the two rounds preceding the task, the opponents were responsible for checking the validity of claims. The teacher did not play a role; true and false was exclusively defined by the students. This independence transferred to the task situation: The students trusted their argumentation and did not require feedback from the teacher once they were convinced they had found the right number. Durand-Guerrier et al. (2012) pointed out how conjecturing can motivate students to look deeper into logical structures. In a game situation, the desire to win can motivate students to find good arguments and make conjectures. In the competitive mode the finding of arguments is practiced, whereas the fictional task promotes the movement from a strategic desire to win towards an internal motivation to cooperate. This cooperative communication situation creates ideal preconditions for argumentation as a procedure.

Argumentation as a product

Arguments are the products of argumentation processes and can be examined from a logical point of view. Habermas (1983) states rules for the logical structure of arguments: No speaker may contradict himself, every speaker who uses a warrant for an inference in one case needs to be willing to use this warrant in analogous cases, and different speakers may not use the same expression with varying meanings.

In most argumentations, the warrants used remain implicit. In everyday interaction we usually assume that our conversation partners share the knowledge from which the warrants arise. In mathematical argumentation, it is common to leave out inferential steps if the reader can easily fill the gaps. For students, however, it is not always obvious which knowledge counts as shared and how to find arguments. In logical games, there is not only a fixed set of rules but also a limited number of outcome possibilities. Analogous cases are easily identified and contradictions are easy to see. Context complexity as described by Douek (2002) is reduced: time and space are irrelevant, the sources of arguments are clearly defined by the game's rules and structure, and frame changes between the abstract rules and the concrete situation are easily undertaken. In the task, a further reduction of complexity is achieved by giving the same situation to all students. This way, communication is facilitated.

Although the game is not directly connected to any mathematical content, the mode of reasoning used is essential for the learning of mathematics. General inference rules are used to deduce hypotheses from the data given on the worksheet. The conclusions the students arrive at are certain as long as we assume that all players act according to the instructions. In his work about proving, Jahnke (2007) has established the dependence

of statements on hypotheses as characteristic for mathematical argument. The analysis of the situation at hand has shown that the warrants applied by the students to create arguments were equivalent to the rules of the game. Furthermore, the available data to arrive at conclusions was limited by the game setting presented to all students. This creates an ideal situation for the development of argumentation.

WHAT IS THERE TO LEARN FROM LOGICAL GAMES?

Proof is an essential component of academic mathematics, and so the products of argumentation have often been in the focus of mathematics education research. However, if we want to take a closer look at the products, we might have to look more closely at what Habermas calls ‘processes’ and ‘procedures’ as well. Habermas’ theory of communicative action does not specifically focus on the mathematics classroom but on how argumentation spontaneously develops in society. If we want to include more students in argumentation, taking a closer look at Habermas’ criteria for when individuals engage in argumentation can be a helpful means.

Logical games may help to establish a situation where force from outside is excluded and a cooperative mode of communication is predominant. In this environment, argumentation can be practiced in a meaningful and motivational way. Especially for students who are not used to argumentation, this presents a good opportunity to develop and practice their reasoning skills. In a game situation, all participants have equal power, rights and duties, and the same limitations seem true for everyone. In the light of social imbalances whose high impact on mathematical argumentation Knipping (2012) has pointed out, games could present one way of overcoming problems.

Another clear advantage of game situations is their clarity about applicable warrants and about the scope of data that can be used as a reference. The concrete and the abstract are tightly linked in the game situation, because the abstract rules guide the argumentation in a concrete situation. The steps from data to conclusion in a logical game, which one student makes, are easily comprehensible for the other participants in the situation. Despite the easy construction of arguments in this structured game context, the conclusions are not obvious. Logical games are often designed so that logical thinking and arguments with several intermediate steps are necessary to arrive at a conclusion. The products arising from these situations are likely to be sophisticated arguments.

Last but not least, the motivation to win a game by producing the cleverest argument creates a positive atmosphere in the classroom. Children are fascinated by games in general, and if these games contain argumentation they may become even more interested in the search for the best argument, which is so typical for the science of mathematics.

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